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Propagation of electromagnetic waves in a plasma

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Taking collisions into account, the problem of propagation of EM waves in a plasma, free from any external electric or magnetic field has been studied classically through the use of Boltzmann equation and Maxwell's field equations following Van Kampen's method of stationary solutions. The dispersion relation is obtained and is discussed in details. It has been found that in the zero temperature region, a strong damping occurs in case of very low-frequency waves In the collisionless case, a type of resonance occurs in case of high frequency waves, the resonant frequency being independent of the temperature of the plasma

INTRODUCTION

Bernstein (1958) studied waves in plasma in details using Laplace transform method of Landau (1946) Pradhan & Misra (1960) studied transverse waves in plasma, taking collisions into account and following Van Kampen (1955) Felderhof (1963) and Varma (1966) also followed Van Kampen to study the same problem in Vlasov-plasma

Here we study the problem of propagation of electromagnetic waves in plasma, taking collisions into account, following Van Kampen (1955) and Varma (1966). The equilibrium distribution function f_0 has been assumed to be isotropic and Maxwellian The collision term for a Lorentzian plasma as given by Desloge & Matthysse (1960) has been included in the Boltzmann equation. Taking f_0 to be Maxwellian removes the limitation of our treatment regarding the Lorentzian nature of plasma and it becomes applicable to non-Lorentzian plasmas as well This has been justified by Mittal & Kaw (1966). The elastic collisions have been assumed to be strong and isotropic, and the collision frequency independent of the electron velocity. The electric field is assumed to be weak and homogeneous These considerations give isotropic picture of a plasma (Ginzburg & Gurevich, 1960) The dispersion relation is derived and discussed in details It is found that under the condition of zero temperature a strong damping occurs in case of very low frequency waves, the damping being mainly due to collisions. In the collisionless case, a type of resonance occurs in case of high frequency waves, the resonant frequency being independent of the temperature T of the plasma.

In our model of plasma, ions are assumed to be stationary providing a uniform neutralizing positive background and the behaviour of electrons is described by the following Boltzmann equation :

$$\frac{\partial f}{\partial t}+V\cdot\frac{\partial f}{\partial r}+\frac{e}{m}\cdot\left\{E(r,t)+\frac{V\times B(r,t)}{c}\right\}\cdot\frac{\partial f}{\partial V}=\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} \qquad \dots \quad (2.1)$$

where E and B are the electric and magnetic field intensities of electromagnetic waves and f is the single-particle distribution function of electrons For linear approximation f can be broken up into two parts as

$$f(r, V, t) = n_0 f_0(V) + f_1(r, V, t)$$
$$f_1 << f_0 \quad \ldots \quad (2.2)$$

where n_0 and f_0 refer to the equilibrium distribution and f_1 is a very small departure from equilibrium For a Lorentzian plasma, we take the following form for collision term (Desloge & Matthysse, 1960)

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = -\nu(f-f_0)+\frac{m}{MV^2}\cdot\frac{\partial}{\partial V}(\nu V^3 f_0)+\frac{kT}{MV^2}\cdot\frac{\partial}{\partial V}\left(\nu V^2\frac{\partial f_0}{\partial V}\right) \quad \ldots \quad (2.3)$$

where ν is the elastic collision frequency of electrons with neutral molecules, M is the mass of a gas molecule, k is the Boltzmann's constant and T is the temperature of the gas

Now substituting equation (23) on the right hand side of equation (2.1) and on linearization and simplification the collisional Botzmann equation is given as

$$\frac{\partial f_1}{\partial t}+V\cdot\frac{\partial f_1}{\partial z}+\frac{en_0}{m}\cdot\left[E(r,t)+\frac{V\times B(r,t)}{c}\right]\frac{\partial f_0}{\partial V}$$
$$=-Ku_\nu f_1+\frac{Ku_\nu\delta}{2n_0}\left[V\cdot\frac{\partial f}{\partial V}+3f+\frac{C_e^2\partial^2 f}{\partial V^2}+\frac{2C_e^2}{V}\frac{\partial f}{\partial V}\right] \quad \ldots \quad (2.4)$$

where $\nu = Ku_\nu$, $\delta = 2m/M$ is the average fraction of energy lost during one elastic collision and $C_e = (kT/m)^{\frac{1}{2}}$ is the thermal velocity of electrons.

From Maxwell's equations we get the following equation for electric field intensity

$$\nabla\times\nabla\times E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}-\frac{4\pi}{c^2}\cdot\frac{\partial j}{\partial t} \quad \ldots \quad (2.5)$$

where j is the current density and it can be measured as

$$j = e\int Vf_1(r, V, t)dV \quad \ldots \quad (2.6)$$

We are interested in the propagation of waves along the Z-axis of the coordinate system We decompose velocity into two components, one along and the other in perpendicular direction to Z-axis, which are denoted as v_z and V. Now equation (2 4) become

$$\frac{\partial f_1(z,V,t)}{\partial t}+v_z\frac{\partial f_1(z,V,t)}{\partial z}+\frac{en_0}{m}\left[E(z,t)+\frac{V\times B(r,t)}{c}\right]\cdot\frac{\partial f_0(V)}{\partial V}$$
$$=-Ku_\nu f_1(z,V,t)+\frac{Ku_\nu\delta}{2}\left[V\cdot\frac{\partial f_0(z,V,t)}{\partial V}+3f_0(z,V,t)\right.$$
$$\left.+C_e^2\frac{\partial^2 f_0(z,V,t)}{\partial^2 V}+\frac{2C_e^2}{V}\frac{\partial f_0(z,V,t)}{\partial V}\right] \quad (2.7)$$

On multiplying equation (2.7) by $V_\perp$ throughout and integrating over $V_\perp$ we get

$$\frac{\partial f_\perp}{\partial t}+v_z\,.\,\frac{\partial f_\perp}{\partial z}-\frac{en_0}{m}E_\perp F(v_z) = -Ku_\nu f_\perp+\frac{Ku_\nu\delta}{2}\Big[v_z F(v_z)$$

$$+3\int F(v_z)dv_z+C_e^2\frac{dF(v_z)}{dv_z}+\frac{2C_e^2}{v_z}F(v_z)\Big] \qquad \dots \quad (2.8)$$

where

$$f_\perp = \int\int V_\perp f_1(z, v_z, V_\perp t)dV_\perp \qquad \dots \quad (2.9)$$

and

$$F(v_z) = \int\int f_0(v_z, V_\perp)dV_\perp$$

We consider the wave vector K along the z-axis and it is taken to be real; then equation (2.5) for transverse waves becomes

$$\nabla^2 E_\perp-\frac{1}{c^2}\frac{\partial^2 E_\perp}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial j}{\partial t} = \frac{4\pi e}{c^2}\frac{\partial}{\partial t}\int f_\perp dv_z \qquad \dots \quad (2.10)$$

Let us suppose that (Ginzburg & Gurevich 1960)

$$f_\perp^{K,u_s}(z, v_z, t) = g^{K,u_s}(v_z)\exp iK(z-u_s t)$$

$$E_\perp^{K,u_s}(z, t) = \frac{u_\perp m(i\omega+Ku_\nu)}{e} \qquad \dots \quad (2.11)$$

where $u_s = u-iu_\nu$, $u = \omega/k$ and $u_\perp$ is the average velocity of electrons along the direction of the electric field. Substituting equations (2.11) in equation (2.8) we get

$$g^{K,u_s}(v_z)\exp iK(z-u_s t) = -\frac{1}{ik}\Big[u_\perp(i\omega+Ku_\nu)n_0\frac{F(v_z)}{(u-v_z)}$$

$$+\frac{Ku_\nu\delta}{2}\Big\{v_z\frac{F(v_z)}{(u-v_z)}+3\frac{\int F(v_z)dv_z}{(u-v_z)}+\frac{C_e^2.dF(v_z)/v_z}{(u-v_z)}$$

$$+\frac{2C_e^2}{v_z}\frac{F(v_z)}{(u-v_z)} \qquad (2.12)$$

and equation (2.10) yields

$$\frac{mu_\perp(i\omega+Ku_\nu)}{e} = -\frac{4\pi ieu_s}{K(u_s^2-c^2)}\int_{-\infty}^{\infty} g(v_z)dv_z\exp iK(z-u_s t) \qquad (2.13)$$

Eliminating $g^{K,u_s}(v_z)$ from equations (2.12) and (2.13) we get the dispersion relation for the electromagnetic waves :

$$\frac{u_s^2-c^2}{u_p^2.u_s} = \int_{-\infty}^{\infty} \frac{F(v_z)}{(u-v_z)}\,dv_z + \frac{Ku_\nu\,\delta}{2u_\perp n_0(i\omega+Ku)}\Bigg[\int_{-\infty}^{\infty}\frac{v_zF(v_z)}{(u-v_z)}dv_z$$

$$+3\int_{-\infty}^{\infty}\frac{F(v_z)dv_z}{(u-v_z)}dv_z+C_e^2\int_{-\infty}^{\infty}\frac{dF(v_z)dv_z}{(u-v_z)}$$

$$+2C_e^2\int_{-\infty}^{\infty}\frac{F(v_z)}{v_z(u-v_z)}dv_z\Bigg] \quad \ldots \quad (2.14)$$

where
$$u_p^2 = \frac{\omega_p^2}{K^2} = \frac{4\pi n_0 e^2}{mK^2}$$

and ω_p is the plasma frequency The above dispersion relation is independent of boundary conditions

In the collisionless zero-temperature case the equation reduces to

$$u^2-c^2 = u_p^2 \quad \ldots \quad (2.15)$$

because $F(v_z)$ behaves as a delta-function If the following approximate form for the collision term is taken

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = -\nu(f-f_0) = -Ku_\nu f_1 \quad \ldots \quad (2.16)$$

then the dispersion relation is

$$\frac{u_s^2-c^2}{u_p^2\,u_s} = \int_{-\infty}^{\infty}\frac{F(v_z)}{(u-v_z)}.\,dv_z \quad \ldots \quad (2.17)$$

Relation (2 17) has been derived and discussed by Varma (1966).

3. Case of Maxwellian Distribution

Here

$$F(v_z) = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}\exp-\left(\frac{mv_z^2}{2kT}\right) \quad \ldots \quad (3.1)$$

Solving the equation (2.14) and neglecting terms of higher order of $(K/\omega)^3$ and $(K/\omega)^4$ we get

$$\frac{(\omega-iKu_\nu)^2-c^2K^2}{\omega_p^2.(\omega-iKu_\nu)} = \frac{1}{\omega}+\frac{Ku_\nu\delta}{2n_0u_\nu(i\omega+Ku_\nu)}\left[\left(\frac{1}{\omega}+\frac{2K}{\omega^2}\right)C_e^2-\frac{C_e}{\omega}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\right] \ldots \quad (3.2)$$

We shall consider equation (3 2) for the cases of low and high frequency propagation.

Low-frequency propagation: Assuming that the propagating frequency ω is much smaller than the collision frequency ($\omega << K u_\nu$), $K^2 c^2 \omega^2 >> 1$ and retaining terms only upto the first order in ω, we get

$$\frac{2iKu_\nu\omega + K^2 u_\nu^2 + c^2 K^2}{\omega_p^2} = \left(\frac{iKu_\nu}{\omega} - 1\right) + \frac{iu_\nu\delta}{2_\perp u n_0}\left[C_e^2\left(\frac{K}{\omega} + \frac{2K^2}{\omega^2}\right) - \frac{C_e K}{\omega}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\right] \quad \ldots \quad (3.3$$

when $\omega \to 0$, equation (3.3) gives

$$\frac{K^2(u_\nu^2 + c^2)}{\omega_p^2} + 1 = \frac{iu_\nu}{u_p} + \frac{iu_\nu\delta}{2u_\perp n_0}\left[C_e^2\left(\frac{1}{u_p} + \frac{2}{u_p^2}\right) - \frac{C_e}{u_p}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\right] \quad \ldots \quad (3.4)$$

where u_p is the phase velocity. In the zero temperature case we get

$$\frac{K^2(u_\nu^2 + c^2)}{\omega_p^2} + 1 = \frac{iu_\nu}{u_p} \quad \ldots \quad (3.5)$$

Since $I_m(u_p) > 0$, the wave is strongly damped by collisions. At finite frequencies ($\omega << \omega_p$) the wave is damped as indicated by the last term on the right hand side of equation (3 3)

High-frequency propagation When the collision frequencies are much smaller than the propagating frequency ($\omega >> K u_\nu$), and $K^2 c^2/\omega^2 << 1$ the equation (3.2) can be approximated as

$$\frac{\omega^2 - 2iKu_\nu\omega}{\omega_p^2} = 1 - \frac{iKu_\nu\delta}{2u_p n_0} \times \left[\left(\frac{1}{\omega} + \frac{2K}{\omega^2}\right) C_e^2 - \frac{\epsilon_c}{\omega}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\right] \quad \ldots \quad (3.6)$$

In the collisionless case, equation (3.6) gives

$$\omega = \omega_p \quad \ldots \quad (3.7)$$

This indicates that resonance occurs in the collisionless case and that the resonant frequency is independent of the temperature T of the plasma

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